



New Modification of Ranked Set Sampling for Estimating Population Mean: Neutrosophic Median Ranked Set Sampling with an Application to Demographic Data

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Abstract

The study addressed the limitations of classical statistical methods when dealing with ambiguous data, emphasizing the importance of adopting neutrosophic statistics as a more effective alternative. Classical methods falter in managing uncertainty inherent in such data, necessitating a shift towards methodologies like neutrosophic statistics. To address this gap, the research introduced a novel sampling approach called “neutrosophic median ranked set sampling” and incorporated neutrosophic estimators tailored for estimating the population mean in the presence of ambiguity. This modification aims to address the inherent challenges associated with estimating the population mean when dealing with neutrosophic data. The methods employed involved modifying traditional ranked set sampling techniques to accommodate neutrosophic data characteristics. Additionally, neutrosophic estimators were developed to leverage auxiliary information within the framework of median-ranked set sampling, enhancing the accuracy of population mean estimation under uncertain conditions. The methods employed involved modifying traditional ranked set sampling techniques to accommodate neutrosophic data characteristics. Bias and mean squared error equations for the suggested estimators were provided, offering insights into their theoretical underpinnings. To illustrate the effectiveness and practical applications of the proposed methodology and estimators, a numerical demonstration and simulation study have been conducted using the R programming language. The key results highlighted the superior performance of the proposed estimators compared to existing alternatives, as demonstrated through comprehensive evaluations based on mean squared error and percentage relative efficiency criteria. The conclusions drawn underscored the effectiveness of the neutrosophic median ranked set sampling approach and suggested estimators in estimating the population mean under conditions of uncertainty, particularly when utilizing neutrosophic auxiliary information and validated real-life applicability. The methodology and estimators presented in the study were shown to yield interval-based results, providing a more realistic representation of uncertainty associated with population parameters. This interval estimation, coupled with minimum mean squared error considerations, enhanced the efficacy of the estimators in determining population mean values. The novelty of the work lies in its introduction of a tailored sampling approach and estimators designed specifically for neutrosophic data, filling a significant gap in the literature. By extending classical statistics to accommodate ambiguity, the study offers a substantial advancement in statistical methodology, particularly in domains where precise data is scarce and uncertainty is prevalent. Furthermore, the empirical validation through numerical demonstrations and simulation studies using the R programming language adds robustness to the proposed methodology and contributes to its practical applicability.

Keywords Neutrosophic statistics · Study variable · Auxiliary variable · Bias · Mean squared error · Percentage relative efficiency

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1 Introduction

Sampling is a crucial practice for a variety of reasons, such as cost and time constraints. In sampling theory, the goal of estimation procedures is to enhance the effectiveness of estimators for population parameters while minimizing sampling errors. To achieve this, auxiliary information is utilized to improve estimator efficiency, and this information can be incorporated at various stages of the process. When highly correlated auxiliary information is not readily available, it can be gathered from previous surveys. Estimation techniques like ratio, product, and regression are commonly employed in this context.

For instance, [25] introduced a modified ratio estimator that incorporates the coefficient of variation of auxiliary information. References [3, 9, 14, 16, 19, 21] have also proposed population parameter estimation methods using auxiliary information. However, our focus here is on ranked set sampling.

Efforts in sampling continually strive for improvements in estimator efficiency, cost-effectiveness, simplicity, and time savings. Ranked set sampling (RSS) offers a superior alternative to simple random sampling (SRS) in various fields, including medicine, agriculture, earth sciences, statistics, and mathematics, especially when measurements are cumbersome, time-consuming, or expensive. The RSS technique was initially described for population mean estimation by [13], and the mathematical theory behind RSS was provided by [35]. Dell and Cullter [7] demonstrated that under both perfect and imperfect ranking scenarios, the mean estimate in RSS remains unbiased.

Numerous researchers, such as [2, 5, 6, 8, 10, 12, 17, 20, 33] have contributed to the field of RSS. For recent work, one can prefer [4, 23, 36, 37].

Only when the data have a symmetric distribution, RSS provide an efficient estimate for the estimation of population parameters; otherwise, the results will be less favorable. To acquire the most accurate parameter estimates, various RSS sampling methods are available. These include double RSS, median ranked set sampling (MRSS), quantile RSS, extreme RSS, a combination of median and extreme RSS, except extreme RSS, and more. For asymmetric one can go for MRSS. In this context, our focus will be on MRSS. [15] originally introduced MRSS, and subsequently, [18] presented a ratio estimator under MRSS. Al-Omari [1] provided ratio estimation under both SRS and MRSS, while [11] contributed a difference-cum-ratio and exponential-type estimator under MRSS.

Classical RSS deals solely with precise data, assuming no uncertainty exists. However, in practice, data can be uncertain and imprecise, containing sets or intervals. To address such situations, fuzzy logic is a valuable tool that handles data with imprecision. When analyzing fuzzy data, fuzzy

statistics are employed, with ambiguous, uncertain, unclear, or imprecise characteristics. Yet, they do not account for the degree of indeterminacy inherent in the data. Neutrosophic statistics, an extension of fuzzy logic, offer a way to measure both indeterminacy and determinate aspects of uncertain or imprecise data. When dealing with data that contains indeterminacy, neutrosophic statistics are employed. Neutrosophic statistics expand upon classical statistics and encompass fuzzy and intuitionistic statistics. Neutrosophy is applicable when observations in a population or sample lack precision, are indeterminate, or are vague. Some examples of neutrosophic data include district-wise water level measurements with intervals, variations in machinery part sizes due to measurement errors, and day-wise temperature measurements resulting in interval-type data.

The concept of neutrosophy was initially introduced by [26–32], and extensive literature on neutrosophic sets, logic, and statistics can be found in his works. In the realm of sampling theory, [34] recently addressed the estimation of population parameters under a neutrosophic environment, introducing neutrosophic ratio-type estimators for population means under SRS. One can also prefer [22, 24] for neutrosophic estimators. However, there has been little focus on neutrosophic RSS (NRSS) for estimating population parameters. References [36, 37] proposed NRSS and a generalized estimator for the computation of the population mean.

Efficiency improvements in sampling framework and estimators are a constant objective in sampling. In this context, we propose an enhanced novel sampling framework neutrosophic median ranked set sampling (NMRSS) and improved NMRSS estimators for population mean estimation, with a particular emphasis on minimizing MSE and enhancing precision.

There is not much emphasis on the problem of estimation when dealing with indeterminate, vague, imprecise, set-type, or interval-type data that is characteristic of neutrosophic data, which is more prevalent in real-world scenarios than crisp data. It is a well-proven theory through a literature review that RSS is an improvement over SRS and MRSS is most efficient when data do not follow symmetric distribution. There is no work in the estimation of parameters using the MRSS technique when there is the presence of errors, unclear, inaccurate, indeterminate, vague, imprecise, set-type, or interval-type data. Hence, our research is novel and significant in the field of sampling where we came with neutrosophic data and SRS and RSS are not efficient sampling techniques to work with. Our work may be more desirable than other work in several real situations like agriculture sciences, mathematical sciences, biological sciences, poultry, business, economics, commerce, social sciences, etc.

The efficacy of the innovative sampling framework, neutrosophic median ranked set Sampling (NMRSS), alongside enhanced NMRSS estimators for population mean estima-

tion, was validated through empirical analysis using both real and simulated population data. Results unequivocally demonstrated the superiority of the recommended estimators over existing ones, showcasing lower MSEs and higher PREs, with t_{p2} emerging as the optimal estimator. As sample sizes and correlation coefficients increased, the MSE decreased and PRE increased for the recommended estimator, indicating its robustness and efficiency. NMRSS estimators surpassed their counterparts NSRS, with RSS proving to be a superior replacement for SRS. Similarly, under NRSS, MRSS outperformed RSS, particularly when data exhibited non-symmetric distributions. Comparative analysis revealed that MSEs and PREs of suggested estimators obtained through classical MRSS fell between those obtained using NMRSS, suggesting the latter's superiority. The study underscored the inadequacy of classical RSS in handling vague or indeterminate data, emphasizing NMRSS as a superior method for estimating uncertain or interval data.

It can be computationally burdensome to analyze neutrosophic data, particularly when processing large datasets with advanced models or algorithms. It can be difficult to interpret analysis based on neutrosophic data, particularly when dealing with contradicting or ambiguous results. Compared to other approaches or even just SRS, RSS implementation can be more complicated. Before sampling, the population must be carefully ranked and sorted, which can take time and require specialized knowledge.

Our study is designed as follows: Sect. 1 presents an introduction, and Sect. 2 outlines motivation, needs, and research gaps. Section 3 outlines the modified and novel sampling method, neutrosophic median ranked set sampling method. Section 4 presents existing NMRSS estimators. Section 5 presents proposed NMRSS estimators, while Sect. 6 presents an application with the help of empirical study using demographic data, and Sect. 7 offers a simulation study using artificial data for different sets of values. Section 8 is dedicated to a discussion, and Sect. 9 covers concluding remarks and future research directions.

2 Motivation, Need and Research Gap

This article's main objective is to introduce a novel approach known as the "neutrosophic median ranked set sampling" for dealing with interval-type or neutrosophic data. Our study focuses on sampling theory, marking the first instance of proposing an MRSS technique tailored to neutrosophic data, along with the development of NMRSS estimators for population mean estimation. This marks a significant step in expanding the field of sampling theory by comparing these estimators with existing neutrosophic methods such as ratio, product, and regression estimators. MRSS is considered a superior alternative to SRS and RSS when data does not fol-

low symmetric distribution, making it an attractive avenue for further exploration in the context of NMRSS.

Several factors drive our exploration of NMRSS and its associated estimators for population parameter estimation. A primary motivation is to introduce MRSS and MRSS estimators in a neutrosophic setting. Previous research in survey sampling has predominantly focused on clear, well-defined data, while classical sampling methods yield precise results, albeit with potential risks of inaccuracies, overestimations, or underestimations. However, classical methods fall short when handling set-type or undetermined data, characteristic of neutrosophic data, which is more prevalent in real-world scenarios than crisp data. Thus, there is a growing need for additional neutrosophic statistical techniques. Traditional statistical approaches are ill-suited to compute accurate estimates of unknown parameters when dealing with indeterminate, vague, imprecise, or interval-type data. Neutrosophic statistics serve as a suitable replacement for classical statistics in such scenarios.

The necessity to close the gap between classical and neutrosophic statistics, and inspired by [36, 37] work, our work introduces a new sampling technique NMRSS, and enhanced NMRSS estimators for population mean estimation. Despite thorough research in the field, we found no prior studies in survey sampling that addressed the estimation of population means in the presence of auxiliary variables using neutrosophic data under MRSS. This research represents a significant step toward filling this gap and contributes to the evolving domain of neutrosophic statistics.

It has been well established by multiple authors that RSS is a more suitable option than SRS when dealing with cumbersome, expensive, or time-consuming measurements and MRSS is a better option than RSS when data does not follow the symmetric distribution. The challenges associated with measurements in a neutrosophic context exacerbate these problems. Therefore, our research introduces an NMRSS method to enhance the accuracy of the population mean estimators in this unique context.

3 Sampling Methodology

Numerous methods can be used to display the neutrosophic observations, and the neutrosophic numbers may include an unknown interval $[a, b]$. We are describing neutrosophic values as $Z_{rssN} \equiv Z_{rssL} + Z_{rssU}I_{rssN}$ with $I_{rssN} \in [I_{rssL}, I_{rssU}]$, N is here to represent the neutrosophic number. Hence, our neutrosophic observations will lie in an interval $Z_{rssN} \in [a, b]$, where 'a' and 'b' denote the neutrosophic data's lower and upper values.

In MRSS, a small subset of randomly chosen population units are measured after they have been ranked solely based on observation or experience. In the context of MRSS,

m independent sets of random samples, each consisting of m units, are drawn from the overall population. Every unit within a set has an equal probability of selection. The members of each random set are then arranged in order based on the characteristics of the auxiliary variable. Then, if the sample size m is even, identify $(\frac{m}{2})$ th smallest ranked unit from the first $(\frac{m}{2})$ sets, and select $(\frac{m+2}{2})$ th smallest ranked unit from the other $(\frac{m}{2})$ sets. In case when the sample size m is odd, identify $(\frac{m+1}{2})$ th smallest ranked unit from all sets for actual quantification. Throughout the process, rm ($=n$) units have been acquired as this cycle is replicated r times.

The method of NMRSS consists of selecting $m_N \in [m_L, m_U]$ bivariate random samples of size $m_N \in [m_L, m_U]$ from a population of size N , and then ranking inside each sample concerning for auxiliary variable $X_N \in [X_L, X_U]$ associated with $Y_N \in [Y_L, Y_U]$. The book by [32] entitled "Introduction to Neutrosophic Statistics" will be the basis for the ranking of the neutrosophic number. To show the process of ranking, we are utilizing here two sets as $X_{1N} \in [X_{1L}, X_{1U}]$ and $X_{2N} \in [X_{2L}, X_{2U}]$, also their mid-points are as $X_{1midN} = [X_{1L} + X_{1U}]/2$ and $X_{2midN} = [X_{2L} + X_{2U}]/2$. The ordering of neutrosophic numbers $m_N \in [m_L, m_U]$ can be done as $X_{1N} \in [X_{1L}, X_{1U}]$ will be less than $X_{2N} \in [X_{2L}, X_{2U}]$ if $X_{1midN} \leq X_{2midN}$, also if both are same that is $X_{1midN} = X_{2midN}$ then we will compare or see by $X_{1L} \leq X_{2L}$. Further, if again $X_{1L} = X_{2L}$ then this implies $X_{1U} = X_{2U}$ and hence $X_{1N} \in [X_{1L}, X_{1U}] = X_{2N} \in [X_{2L}, X_{2U}]$, so the neutrosophic number ranking will be carried out in this manner.

In the whole NMRSS structure if $m_N \in [m_L, m_U]$ is even, we count $(\frac{m_N}{2})$ th smallest rank unit of the first $(\frac{m_N}{2})$ data of set size $m_N \in [m_L, m_U]$, for the first $(\frac{m_N}{2})$ measurement units in the entire NMRSS structure, and then we scrap the remaining units. In a similar manner, we count the $(\frac{m_N+2}{2})$ th smallest observation from the remaining $(\frac{m_N}{2})$ data set for the remaining $(\frac{m_N}{2})$ measurement units and discard the remaining observations. If $m_N \in [m_L, m_U]$ is odd, we select $(\frac{m_N+1}{2})$ th smallest rank unit from all $m_N \in [m_L, m_U]$ sets of size $m_N \in [m_L, m_U]$. This process counts the total $m_N \in [m_L, m_U]$ neutrosophic bivariate units. After r cycles of these steps, the total $n_N = m_N r \in [n_L, n_U]$ bivariate NMRSS units are obtained.

Here, we have set sample NMRSSo for an odd set size and NMRSSe for an even set size. For j th cycle MRSSo is denoted by $(X_{1(\frac{m_N+1}{2})j}, Y_{1[\frac{m_N+1}{2}]j}), (X_{2(\frac{m_N+1}{2})j}, Y_{2[\frac{m_N+1}{2}]j}), \dots, (X_{m_N(\frac{m_N+1}{2})j}, Y_{m_N[\frac{m_N+1}{2}]j})$; ($j = 1, 2, \dots, r$). Let the sample means of X_N and Y_N are, respectively, as

$$\bar{x}_{(o)N} = \frac{1}{rm_N} \sum_{j=1}^r \sum_{i=1}^{m_N} x_{i(\frac{m_N+1}{2})j} \quad (1)$$

$$\bar{y}_{(o)N} = \frac{1}{rm_N} \sum_{j=1}^r \sum_{i=1}^{m_N} y_{i[\frac{m_N+1}{2}]j} \quad (2)$$

and their corresponding variances are given as

$$\begin{aligned} \text{Var}(\bar{x}_{(o)N}) &= \text{Var} \left(\frac{1}{rm_N} \sum_{j=1}^r \sum_{i=1}^{m_N} x_{i(\frac{m_N+1}{2})j} \right) \\ &= \frac{1}{rm_N} \sigma_{x(\frac{m_N+1}{2})}^2 \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Var}(\bar{y}_{(o)N}) &= \text{Var} \left(\frac{1}{rm_N} \sum_{j=1}^r \sum_{i=1}^{m_N} y_{i[\frac{m_N+1}{2}]j} \right) \\ &= \frac{1}{rm_N} \sigma_{y[\frac{m_N+1}{2}]}^2. \end{aligned} \quad (4)$$

Similarly, for j th cycle NMRSSe is denoted by $(X_{1(\frac{m_N}{2})j}, Y_{1[\frac{m_N}{2}]j}), (X_{2(\frac{m_N}{2})j}, Y_{2[\frac{m_N}{2}]j}), \dots, (X_{\frac{m_N}{2}+1(\frac{m_N}{2})j}, Y_{\frac{m_N}{2}+1[\frac{m_N}{2}]j}), \dots, (X_{m(\frac{m_N}{2})j}, Y_{m[\frac{m_N}{2}]j})$. Again, let the sample means of X_N and Y_N are, respectively, as

$$\bar{x}_{(e)N} = \frac{1}{rm_N} \sum_{j=1}^r \left(\sum_{i=1}^{\frac{m_N}{2}} x_{i(\frac{m_N}{2})j} + \sum_{i=\frac{m_N}{2}+1}^{m_N} x_{i(\frac{m_N+2}{2})j} \right) \quad (5)$$

$$\bar{y}_{(e)N} = \frac{1}{rm_N} \sum_{j=1}^r \left(\sum_{i=1}^{\frac{m_N}{2}} y_{i[\frac{m_N}{2}]j} + \sum_{i=\frac{m_N}{2}+1}^{m_N} y_{i[\frac{m_N+2}{2}]j} \right) \quad (6)$$

and their corresponding variances are given as

$$\begin{aligned} \text{Var}(\bar{x}_{(e)N}) &= \text{Var} \left(\frac{1}{rm_N} \sum_{j=1}^r \left(\sum_{i=1}^{\frac{m_N}{2}} x_{i(\frac{m_N}{2})j} + \sum_{i=\frac{m_N}{2}+1}^{m_N} x_{i(\frac{m_N+2}{2})j} \right) \right) \\ &= \frac{1}{2rm_N} \left(\sigma_{x(\frac{m_N}{2})}^2 + \sigma_{x(\frac{m_N+2}{2})}^2 \right) \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Var}(\bar{y}_{(e)N}) &= \text{Var} \left(\frac{1}{rm_N} \sum_{j=1}^r \left(\sum_{i=1}^{\frac{m_N}{2}} y_{i[\frac{m_N}{2}]j} + \sum_{i=\frac{m_N}{2}+1}^{m_N} y_{i[\frac{m_N+2}{2}]j} \right) \right) \\ &= \frac{1}{2rm_N} \left(\sigma_{y[\frac{m_N}{2}]}^2 + \sigma_{y[\frac{m_N+2}{2}]}^2 \right) \end{aligned} \quad (8)$$

Consider a neutrosophic random sample of size $n_N \in [n_L, n_U]$ using NMRSS, which is acquired from a finite

population of N units ($U_1, U_2, \dots, U_N$). The neutrosophic study and auxiliary variable are $Y_N \in [Y_L, Y_U]$ and $X_N \in [X_L, X_U]$. Let $\bar{y}_{[n]N} \in [\bar{y}_{[n]L}, \bar{y}_{[n]U}]$ and $\bar{x}_{(n)N} \in [\bar{x}_{(n)L}, \bar{x}_{(n)U}]$ be the sample means of the neutrosophic study and auxiliary variables respectively, and also, let $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$ be the population means of the neutrosophic study and auxiliary variables, respectively. The correlation coefficient between both neutrosophic study and auxiliary variables is $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$.

Let the neutrosophic mean error terms are $\epsilon_{0N} \in [\epsilon_{0L}, \epsilon_{0U}]$ and $\epsilon_{1N} \in [\epsilon_{1L}, \epsilon_{1U}]$. To obtain the bias and MSE of the estimators, we write

$$\begin{aligned}\bar{y}_{[n]N} &= \bar{Y}_N(1 + \epsilon_{0N}), \bar{x}_{(n)N} = \bar{X}_N(1 + \epsilon_{1N}) \\ E(\epsilon_{0N}^2) &= \frac{\text{Var}(\bar{y}_{[n]N})}{\bar{Y}_N^2} = V_{yrN} \\ E(\epsilon_{1N}^2) &= \frac{\text{Var}(\bar{x}_{(n)N})}{\bar{X}_N^2} = V_{xrN} \\ E(\epsilon_{0N}, \epsilon_{1N}) &= \frac{\text{Cov}(\bar{y}_{[n]N}, \bar{x}_{(n)N})}{\bar{Y}_N \bar{X}_N} = V_{yxrN} \\ e_{0N}^2 &\in [e_{0L}^2, e_{0U}^2]; e_{1N}^2 \in [e_{1L}^2, e_{1U}^2]; e_{0N}e_{1N} \in [e_{0L}e_{1L}, e_{0U}e_{1U}]; \\ V_{yrN} &\in [V_{yrL}, V_{yrU}]; V_{xrN} \in [V_{xrL}, V_{xrU}]; V_{yxrN} \in [V_{yxrL}, V_{yxrU}].\end{aligned}$$

4 Existing Estimators

The usual unbiased estimator for the population mean $\bar{Y}$ using NMRSS technique is given by

$$\bar{y}_{[n]N} = \frac{1}{n_N} \sum_{i=1}^{n_N} y_{[i]N}. \quad (9)$$

The variance of the estimator $\bar{y}_{[n]N}$ is given by

$$\text{var}(\bar{y}_{[n]N}) = \bar{Y}_N^2 V_{yrN}. \quad (10)$$

The ratio estimator under NMRSS for the population mean $\bar{Y}$

$$\bar{y}_{rN} = \bar{y}_{[n]N} \left(\frac{\bar{X}_N}{\bar{x}_{(n)N}} \right). \quad (11)$$

The MSE of the estimator $\bar{y}_{rN}$ is given by

$$\text{MSE}(\bar{y}_{rN}) = \bar{Y}_N^2 (V_{yrN} + V_{xrN} - 2V_{yxrN}). \quad (12)$$

The product estimator under NMRSS for the population mean $\bar{Y}$

$$\bar{y}_{pN} = \bar{y}_{[n]N} \left(\frac{\bar{x}_{(n)N}}{\bar{X}_N} \right). \quad (13)$$

The MSE of the estimator $\bar{y}_{pN}$ is given by

$$\text{MSE}(\bar{y}_{rN}) = \bar{Y}_N^2 (V_{yrN} + V_{xrN} + 2V_{yxrN}). \quad (14)$$

The regression estimator under NMRSS for the population mean $\bar{Y}$

$$\bar{y}_{\text{reg}N} = \bar{y}_{[n]N} + \beta(\bar{X}_N - \bar{x}_{(n)N}). \quad (15)$$

The MSE of the estimator $\bar{y}_{\text{reg}N}$ is given by

$$\text{MSE}(\bar{y}_{\text{reg}N}) = \bar{Y}_N^2 \left(V_{yrN} - \frac{V_{yxrN}^2}{V_{xrN}} \right). \quad (16)$$

The exponential ratio-type estimator under NMRSS for the population mean $\bar{Y}$ is represented as

$$\bar{y}_{\text{exp}N} = \bar{y}_{[n]N} \exp \left(\frac{\bar{X}_N - \bar{x}_{(n)N}}{\bar{X}_N + \bar{x}_{(n)N}} \right). \quad (17)$$

The MSE of the estimator $\bar{y}_{\text{exp}N}$ is given by

$$\text{MSE}(\bar{y}_{\text{exp}N}) = \bar{Y}_N^2 \left(V_{yrN} + \frac{V_{yrN}}{4} - V_{yxrN} \right). \quad (18)$$

The exponential product type estimator under NMRSS for the population mean $\bar{Y}$ is represented as

$$\bar{y}_{\text{exp}pN} = \bar{y}_{[n]N} \exp \left(\frac{\bar{x}_{(n)N} - \bar{X}_N}{\bar{x}_{(n)N} + \bar{X}_N} \right). \quad (19)$$

The MSE of the estimator $\bar{y}_{\text{exp}pN}$ is given by

$$\text{MSE}(\bar{y}_{\text{exp}pN}) = \bar{Y}_N^2 \left(V_{yrN} + \frac{V_{yrN}}{4} + V_{yxrN} \right). \quad (20)$$

5 Proposed Estimators

No single estimator is universally effective in all situations. Hence, it is preferable to employ estimators that yield minimal MSE and high precision. The objective of this section is to formulate estimators that demonstrate effective performance across a broader range of circumstances. In pursuit of this goal, we have embraced the [14] estimator within the framework of NMRSS and introduced two novel estimators for the mean of a finite population under NMRSS, utilizing auxiliary variables.

$$(1) P_{1N} = \bar{y}_{[n]N} (g_{1N} + 1) + g_{2N} \log \left(\frac{\bar{x}_{(n)N}}{\bar{X}_N} \right), \quad (21)$$

where the constants g_{1N} and g_{2N} ensure that the MSE of the estimators is minimized.

When we express the estimator P_{1N} from Eq. (21) into terms of ϵ 's, we obtain

$$P_{1N} = \bar{Y}_N (1 + \epsilon_{01}) \\ (g_{1N} + 1) + g_{2N} \log \left(\frac{\bar{X}_N (1 + \epsilon_{1N})}{\bar{X}_N} \right). \quad (22)$$

Taking expectations by focusing on first-order approximation, we obtain MSE,

$$\text{MSE}(P_{1N}) = \bar{Y}_N^2 V_{yrN} + g_{1N}^2 A_{1N} + g_{2N}^2 B_{1N} \\ + 2g_{1N} C_{1N} + 2g_{2N} D_{1N} + 2g_{1N} g_{2N} E_{1N}, \quad (23)$$

where

$$A_{1N} = \bar{Y}_N^2 (1 + V_{yrN}) \\ B_{1N} = V_{xrN} \\ C_{1N} = \bar{Y}_N^2 V_{yrN} \\ D_{1N} = \bar{Y}_N V_{yxrN} \\ E_{1N} = \bar{Y}_N \left(V_{yxrN} - \frac{1}{2} V_{xrN} \right).$$

To find out the minimum MSE for P_{1N} , we partially differentiate equation (23) w.r.t. g_{1N} & g_{2N} and equating to zero, we obtain

$$g_{1N}^* = \frac{B_{1N} C_{1N} - D_{1N} E_{1N}}{E_{1N}^2 - A_{1N} B_{1N}} \quad (24)$$

$$g_{2N}^* = \frac{A_{1N} D_{1N} - C_{1N} E_{1N}}{E_{1N}^2 - A_{1N} B_{1N}}. \quad (25)$$

By putting the optimum value of g_{1N} & g_{2N} in the Eq. (23), we can determine the minimum MSE of P_{1N} as

$$\text{MinMSE} \\ = C_{1N} + \frac{B_{1N} C_{1N}^2 + A_{1N} D_{1N}^2 - 2C_{1N} D_{1N} E_{1N}}{E_{1N}^2 - A_{1N} B_{1N}} \quad (26)$$

$$(2) P_{2N} = g_{3N} \bar{Y}_{[n]N} + g_{4N} \exp \left(\frac{\bar{X}_N - \bar{x}_{(n)N}}{\bar{X}_N + \bar{x}_{(n)N}} \right) \\ \left(1 + \log \frac{\bar{x}_{(n)N}}{\bar{X}_N} \right). \quad (27)$$

When we express the estimator P_{2N} from Eq. (27) into terms of ϵ 's, we obtain

$$P_{2N} = g_{3N} \bar{Y}_N (1 + \epsilon_{0N}) + g_{4N} \exp \left(\frac{-\epsilon_{1N}}{2 + \epsilon_{1N}} \right)$$

$$\times (1 + \log (1 + \epsilon_{1N})) \quad (28)$$

$$P_{2N} - \bar{Y}_N = (g_{3N} - 1) \bar{Y}_N + g_{3N} \bar{Y}_N \epsilon_{0N} \\ + g_{4N} \left(1 + \frac{\epsilon_{1N}}{2} - \frac{5\epsilon_{1N}^2}{8} \right) \quad (29)$$

$$\text{Bias}(P_{2N}) = \bar{Y}_N (g_{3N} - 1) + g_{4N} \left[1 - \frac{5}{8} V_{xrN} \right]. \quad (30)$$

Case 1: If sum of weights is fixed ($g_{3N} + g_{4N} = 1$)

The MSE of the estimator P_{2N} is shown as

$$\text{MSE}(P_{2N}) = \bar{Y}_N^2 \left[V_{yrN} + g_{4N}^2 V_{xrN} - 2g_{4N} V_{yxrN} \right]. \quad (31)$$

To find out the minimum value of MSE for P_{2N} , we partially differentiate equation (31) w.r.t. g_{4N} , and equating to zero we get

$$g_{4N}^* = \frac{V_{yxrN}}{V_{xrN}}. \quad (32)$$

By putting the optimum value of g_{4N} in Eq. (31), we can determine the minimum MSE of P_{2N} as

$$\text{MinMSE} = \bar{Y}_N^2 \left(V_{yrN} - \frac{V_{yxrN}^2}{V_{xrN}} \right). \quad (33)$$

Case 2: If the sum of weights is adjustable ($g_{3N} + g_{4N} \neq 1$)

$$P_{2N} - \bar{Y}_N = (g_{3N} - 1) \bar{Y}_N + g_{3N} \bar{Y}_N \epsilon_{0N} + g_{4N} \\ \left(1 + \frac{\epsilon_{1N}}{2} - \frac{5\epsilon_{1N}^2}{8} \right). \quad (34)$$

Squaring on both sides we get

$$(P_{2N} - \bar{Y}_N)^2 \\ = \bar{Y}_N^2 + \bar{Y}_N^2 g_{3N}^2 (1 + \epsilon_{01}^2) + g_{4N}^2 (1 - \epsilon_{1N}^2) \\ - 2g_{3N} \bar{Y}_N^2 - 2g_{4N} \bar{Y}_N \left(1 - \frac{5\epsilon_{1N}^2}{8} \right) \\ + 2g_{3N} g_{4N} \bar{Y}_N \left(1 - \frac{5\epsilon_{1N}^2}{8} + \frac{\epsilon_{0N} \epsilon_{1N}}{2} \right). \quad (35)$$

Taking expectations by focusing on first-order approximation, we obtain MSE,

$$\text{MSE}(P_{2N}) = \bar{Y}_N^2 + g_{3N}^2 A_{2N} + g_{4N}^2 B_{2N} - 2g_{3N} C_{2N} \\ - 2g_{4N} D_{2N} + 2g_{3N} g_{4N} E_{2N}, \quad (36)$$

where

$$A_{2N} = \bar{Y}_N^2 (1 + V_{yrN})$$

$$B_{2N} = 1 - V_{xrN}$$

$$C_{2N} = \bar{Y}_N^2$$

$$D_{2N} = \bar{Y}_N \left(1 - \frac{5}{8} V_{xrN} \right)$$

$$E_{2N} = \bar{Y}_N \left(1 - \frac{5}{8} V_{xrN} + \frac{1}{2} V_{yxrN} \right).$$

To find out the minimum MSE for P_{2N} , we partially differentiate equation (36) w.r.t. g_{3N} & g_{4N} and equating to zero we obtain

$$g_{3N}^* = \frac{B_{2N}C_{2N} - D_{2N}E_{2N}}{A_{2N}B_{2N} - E_{2N}^2} \quad (37)$$

$$g_{4N}^* = \frac{A_{2N}D_{2N} - C_{2N}E_{2N}}{A_{2N}B_{2N} - E_{2N}^2}. \quad (38)$$

By putting the optimum value of g_{3N} & g_{4N} in the Eq. (36), we can determine the minimum MSE of P_{2N} as

$$\text{MinMSE} = C_{2N} + \frac{B_{2N}C_{2N}^2 + A_{2N}D_{2N}^2 - 2C_{2N}D_{2N}E_{2N}}{E_{2N}^2 - A_{2N}B_{2N}}. \quad (39)$$

$$(3) P_{3N} = g_{5N}\bar{Y}_{[n]N} + g_{6N} \left(\frac{\bar{X}_N}{\bar{x}_{(n)N}} \right) \exp \left(\frac{\bar{X}_N - \bar{x}_{(n)N}}{\bar{X}_N + \bar{x}_{(n)N}} \right). \quad (40)$$

When we express the estimator P_{3N} from Eq. (40) into terms of $\epsilon's$, we obtain

$$P_{3N} = g_{5N}\bar{Y}_N (1 + \epsilon_{0N}) + g_{6N} \times (1 + \epsilon_{1N})^{-1} \exp \left(\frac{-\epsilon_1}{2 + \epsilon_{1N}} \right) \quad (41)$$

$$P_{3N} - \bar{Y}_N = (g_{5N} - 1)\bar{Y}_N + g_{5N}\bar{Y}_N\epsilon_{01} + g_{6N} \left(1 - \frac{3\epsilon_{1N}}{2} + \frac{15\epsilon_{1N}^2}{8} \right) \quad (42)$$

$$\text{Bias}(P_{3N}) = \bar{Y}_N (g_{5N} - 1) + g_{6N} \left[1 + \frac{15}{8} V_{xrN} \right]. \quad (43)$$

Case 1: If sum of weights is fixed ($g_{5N} + g_{6N} = 1$)

The MSE of the estimator P_{3N} is shown as

$$\text{MSE}(P_{3N}) = \bar{Y}_N^2 \left[V_{yrN} + g_{6N}^2 V_{xrN} - 2g_{6N} V_{yxrN} \right]. \quad (44)$$

To find out the minimum value of MSE for P_{3N} , we partially differentiate equation (44) w.r.t. g_{6N} and equating to zero, we get

$$g_{6N}^* = \frac{V_{yxrN}}{V_{xrN}}. \quad (45)$$

By putting the optimum value of g_{6N} in the Eq. (44), we can determine the minimum MSE of P_{3N} as

$$\text{MinMSE} = \bar{Y}_N^2 \left(V_{yrN} - \frac{V_{yxrN}^2}{V_{xrN}} \right). \quad (46)$$

Case 2: If the sum of weights is adjustable ($g_{5N} + g_{6N} \neq 1$)

$$P_{3N} - \bar{Y}_N = (g_{5N} - 1)\bar{Y}_N + g_{5N}\bar{Y}_N\epsilon_{0N} + g_{6N} \left(1 - \frac{3\epsilon_{1N}}{2} + \frac{15\epsilon_{1N}^2}{8} \right). \quad (47)$$

Squaring on both sides we get

$$\begin{aligned} (P_{3N} - \bar{Y}_N)^2 &= \bar{Y}_N^2 + \bar{Y}_N^2 g_{5N}^2 (1 + \epsilon_{0N}^2) + g_{6N}^2 \left(1 + 6\epsilon_{1N}^2 \right) \\ &\quad - 2g_{5N}\bar{Y}_N^2 - 2g_{6N}\bar{Y}_N \left(1 + \frac{15\epsilon_{1N}^2}{8} \right) \\ &\quad + 2g_{5N}g_{6N}\bar{Y}_N \left(1 + \frac{15\epsilon_{1N}^2}{8} - \frac{3\epsilon_{0N}\epsilon_{1N}}{2} \right). \end{aligned} \quad (48)$$

Taking expectations by focusing on first-order approximation, we obtain MSE,

$$\text{MSE}(P_{3N}) = \bar{Y}_N^2 + g_{5N}^2 A_{3N} + g_{6N}^2 B_{3N} - 2g_{5N}C_{3N} - 2g_{6N}D_{3N} + 2g_{5N}g_{6N}E_{3N}, \quad (49)$$

where

$$A_{3N} = \bar{Y}_N^2 (1 + V_{yrN})$$

$$B_{3N} = 1 + 6V_{xrN}$$

$$C_{3N} = \bar{Y}_N^2$$

$$D_{3N} = \bar{Y}_N \left(1 + \frac{15}{8} V_{xrN} \right)$$

$$E_{3N} = \bar{Y}_N \left(1 + \frac{15}{8} V_{xrN} - \frac{3}{2} V_{yxrN} \right).$$

To find out the minimum MSE for P_{3N} , we partially differentiate equation (49) w.r.t. g_{5N} & g_{6N} and equating to zero, we get

$$g_{5N}^* = \frac{B_{3N}C_{3N} - D_{3N}E_{3N}}{A_{3N}B_{3N} - E_{3N}^2} \quad (50)$$

$$g_{6N}^* = \frac{A_{3N}D_{3N} - C_{3N}E_{3N}}{A_{3N}B_{3N} - E_{3N}^2}. \quad (51)$$

By putting the optimum value of g_{5N} & g_{6N} in the Eq. (49), we can determine the minimum MSE of P_{3N} as

$$\text{MinMSE} = C_{3N} + \frac{B_{3N}C_{3N}^2 + A_{3N}D_{3N}^2 - 2C_{3N}D_{3N}E_{3N}}{E_{3N}^2 - A_{3N}B_{3N}} \quad (52)$$

where $P_{iN} \in [P_{iL}, P_{iU}]$; $i = 1, 2, 3$, $A_{iN} \in [A_{iL}, A_{iU}]$; $i = 1, 2, 3$, $B_{iN} \in [B_{iL}, B_{iU}]$; $i = 1, 2, 3$

$C_{iN} \in [C_{iL}, C_{iU}]$; $i = 1, 2, 3$, $D_{iN} \in [D_{iL}, D_{iU}]$; $i = 1, 2, 3$, $E_{iN} \in [E_{iL}, E_{iU}]$; $i = 1, 2, 3$.

6 Numerical Illustrations

In this section, we compare the performance of the suggested estimators to the other existing estimators considered in this paper. We have taken real-life natural growth rate data from the sample registration system (2020). The data mentioned in the sample registration system (2020) have four neutrosophic variables for every state, but in our research, we use birth rate vs natural growth rate. Here, birth rate is the neutrosophic auxiliary variable $X_N \in [X_L, X_U]$ and natural growth rate is a neutrosophic study variable $Y_N \in [Y_L, Y_U]$.

To obtain the relative performance of proposed estimators under NMRSS, we have taken two cases of odd and even set sizes. Further, in odd case we have drawn total $n_N = m_N r = 12$ samples with set size $m_N = [3, 3]$ and replication $r = 4$ and for even case we have drawn total $n_N = m_N r = 16$ samples with set size $m_N = [4, 4]$ with replication $r = 4$ from the given population of size 36 by utilizing the method of NMRSS. The NMRSS sample for the study and auxiliary variables are drawn simultaneously using the NMRSS technique outlined in Sect. 3. The following is a definition of the percent relative efficiency (PRE) formula:

$$\text{PRE (Estimators)} = \frac{\text{MSE}(\bar{y}_{[n]N})}{\text{MSE}(\text{estimator})} \times 100. \quad (53)$$

7 Simulation Studies

We verify the effectiveness of the suggested estimator through simulation studies with other existing estimators like the conventional, ratio, regression estimator, etc. This is done via the following steps

1. It is well known that a neutrosophic normal distribution (NND) will be followed by neutrosophic random variables (NRV), i.e. $(X_N, Y_N) \sim NN[(\mu_{xN}, \sigma_{xN}^2), (\mu_{yN}, \sigma_{yN}^2)]$,

$X_N \in [X_L, X_U]$, $Y_N \in [Y_L, Y_U]$, $\mu_{xN} \in [(\mu_{xL}, \mu_{xU})]$, $\mu_{yN} \in [(\mu_{yL}, \mu_{yU})]$, $\sigma_{xN}^2 \in [\sigma_{xL}^2, \sigma_{xU}^2]$, $\sigma_{yN}^2 \in [\sigma_{yL}^2, \sigma_{yU}^2]$.

4-variate random observations with a size of $N = 1000$ have been produced from a 4-variate normal distribution with mean $(\mu_{xL}, \mu_{yL}, \mu_{xU}, \mu_{yU}) = (50, 50, 60, 60)$ and covariance

$$\text{matrix} \begin{bmatrix} \sigma_{xL}^2 & \rho_{xyL}\sigma_{xL}\sigma_{yL} & 0 & 0 \\ \rho_{xyL}\sigma_{xL}\sigma_{yL} & \sigma_{yL}^2 & 0 & 0 \\ 0 & 0 & \sigma_{xU}^2 & \rho_{xyU}\sigma_{xU}\sigma_{yU} \\ 0 & 0 & \rho_{xyU}\sigma_{xU}\sigma_{yU} & \sigma_{yU}^2 \end{bmatrix},$$

where we have $\sigma_{xL}^2 = 100$, $\sigma_{yL}^2 = 100$, $\sigma_{xU}^2 = 121$, $\sigma_{yU}^2 = 121$.

2. For this $N = 1000$ simulated population, the parameters were computed.

3. A sample of size n with $m_N = 3$ for odd case and $m_N = 4$ for even case with $r = 4$ has been selected from this simulated population.

4. To find the MSE of each estimator under study, use the sample data.

5. To get MSEs, the entire step 3–4 process was repeated 10,000 times. The MSE of each estimator of the population mean is the average of the 10,000 values that were obtained.

6. The formula has been used to find each estimator's PRE with regard to $\bar{y}_{[n]N}$.

8 Discussion

The study established mathematical expressions for the very first NMRSS estimators, approximating up to the first order. Subsequently, to examine the properties of the proposed NMRSS estimators, numerical illustrations and simulation studies were conducted. The former used real-world natural growth rate data, while the latter involved an artificial neutrosophic dataset with varying correlation coefficients and sample sizes. The results were encapsulated in Tables 2, 3, 4, and 5, showcasing MSEs and PREs for both existing and proposed NMRSS estimators for odd and even cases. The PREs of the NMRSS estimators were calculated over estimators under NSRS and NRSS, and these outcomes are shown in Tables 6 and 7.

In Tables 2 and 3, the MSEs of the existing and suggested estimators are given along with PRE for odd and even cases respectively. The superiority of the suggested NMRSS estimators over the existing ones is displayed in Tables 2 and 3 in the bolded text. We also see that the MSE and PRE of the recommended estimator are lesser and higher than those of the other existing estimators. The tables make this clear that recommended estimators outperformed existing ones, offering lower MSEs and higher PREs, and t_{p2} being the best estimator.

Table 1 The data of natural growth rate as per SRS 2020

State	BRI	BRu	NGRI	NGRu	State	BRI	BRu	NGRI	NGRu
Andhra Pradesh	15	16	9	10.1	Uttar Pradesh	22.1	26.1	19.3	16.7
Assam	14.3	21.9	8.9	15.5	Uttarakhand	15.6	17	10.5	10.3
Bihar	21	26.2	15.7	20.7	West Bengal	11.2	16.1	10.8	5.4
Chhattisgarh	17.3	23.4	11	15	Arunachal Pradesh	15	17.8	11.8	10.6
NCT of Delhi	14.1	15.5	10.6	11.4	Goa	11.7	12.4	6.9	5.3
Gujrat	17.1	21.1	12	15.1	Himachal Pradesh	10	15.7	8.7	5.6
Haryana	17.7	21.2	12.3	14.7	Manipur	12.8	13.5	9.5	8
Jammu & Kashmir	11.1	16.1	7	11.3	Meghalaya	12.9	25.1	19.6	8.5
Jharkhand	17.6	23.4	13.1	17.9	Mizoram	11.7	16.8	13	7.1
Karnataka	15	17.5	10.2	10.5	Nagaland	11.8	12.9	9	8.4
Kerala	13.1	13.3	6.1	6.3	Sikkim	14	18.2	14.5	9.7
Madhya Pradesh	18.8	26	13.1	19.2	Tripura	10.7	13.4	8	4.2
Maharashtra	14.6	15.3	9.1	10.1	Andaman & Nicobar	10	11.5	5.4	4.7
Odisha	13.1	18.7	6.6	11.2	Chandigarh	12.8	18.1	14	9
Punjab	13.6	14.9	6.6	7.9	Dadar Nagar Haveli	18	21.4	18.1	13.3
Rajasthan	20.8	24.4	15.7	18.6	Ladakh	10.8	15.2	10	6.5
Tamil Nadu	13.6	14	6.8	8.5	Lakshadweep	13.1	19.9	12.7	8.1
Telangana	15.9	16.8	9.6	11.7	Puducherry	13.1	13.1	7	5.6

Table 2 The MSE and PRE of the estimators for odd case

Estimators	MSE	PRE
$\bar{y}_{[n]N}$	[0.351545, 1.209508]	[100, 100]
$\bar{y}_{rN}$	[0.110387, 0.200354]	[318, 603]
$\bar{y}_{pN}$	[0.849801, 3.365292]	[35, 41]
$\bar{y}_{regN}$	[0.085726, 0.117522]	[410, 1029]
$\bar{y}_{exprN}$	[0.198829, 0.561602]	[176, 215]
$\bar{y}_{expN}$	[0.568536, 2.144071]	[56, 61]
t_{p1}	[0.085315, 0.115937]	[412, 1043]
t_{p2}	[0.013749, 0.028534]	[2556, 4238]
t_{p3}	[0.020461, 0.029224]	[1718, 4138]

Table 3 The MSE and PRE of the estimators for even case

Estimators	MSE	PRE
$\bar{y}_{[n]N}$	[0.402361, 0.760615]	[100, 100]
$\bar{y}_{rN}$	[0.13007, 0.203868]	[309, 373]
$\bar{y}_{pN}$	[0.918725, 1.843483]	[41, 43]
$\bar{y}_{regN}$	[0.083821, 0.121898]	[480, 623]
$\bar{y}_{exprN}$	[0.235706, 0.416477]	[170, 182]
$\bar{y}_{expN}$	[0.630034, 1.236284]	[61, 63]
t_{p1}	[0.083418, 0.121109]	[482, 628]
t_{p2}	[0.010757, 0.018951]	[3740, 4013]
t_{p3}	[0.017894, 0.027317]	[2248, 2784]

Similarly, in Tables 4 and 5, the MSEs of the recommended and existing estimators are given along with PRE through a simulation study based on artificial neutrosophic data for different values of the correlation coefficient and different sample sizes. Like Tables 2 and 3, the superiority of the suggested NMRSS estimators over the existing ones is displayed in Tables 4 and 5. We also see that the MSEs and PREs of the recommended estimators are lesser and higher, respectively than those of other existing estimators. Hence, Tables 4 and 5 mirrored these findings, with the proposed estimators continuing to outshine existing ones, demonstrating lower MSEs and higher PREs in the simulation study too.

Tables 4 and 5 show that with the increase in sample sizes and correlation coefficients, the MSE decreases, and the PRE increases for the recommended estimator. Therefore, under NMRSS, the suggested estimators exhibit sensitivity similar to that of classical ranked set sampling.

Table 6 features PRE values of the proposed NMRSS estimators compared to NSRS counterparts. We observe from Table 6 that all PRE values exceeded 100 indicating that all the NMRSS estimators are superior to corresponding estimators under NSRS, as RSS is the best replacement for SRS.

Table 7 features PRE values of the proposed NMRSS estimators over NRSS counterparts. We see from Table 7 that all PRE values exceeded 100, indicating that all the NMRSS estimators are superior to corresponding estimators under NRSS, as MRSS is the best replacement for RSS when data do not follow the symmetric distribution.

Table 4 MSEs and PREs of the recommended and existing estimators under NMRSS for odd case

Estimators	$\rho = 0.9$		$\rho = 0.8$	
	MSE	PRE	MSE	PRE
$\bar{y}_{[n]N}$	[25.8558, 66.8489]	[100, 100]	[29.1643, 63.5223]	[100, 100]
$\bar{y}_{rN}$	[9.4335, 13.6782]	[274, 489]	[17.2001, 26.2012]	[170, 242]
$\bar{y}_{pN}$	[82.3835, 248.6207]	[27, 31]	[83.7130, 234.4552]	[27, 35]
$\bar{y}_{regN}$	[8.3641, 11.7963]	[309, 567]	[14.4049, 20.4974]	[202, 310]
$\bar{y}_{exprN}$	[12.6315, 24.1884]	[205, 276]	[17.8592, 28.1602]	[163, 226]
$\bar{y}_{exppN}$	[49.1065, 141.6597]	[47, 53]	[51.1156, 132.2873]	[48, 57]
t_{p1}	[8.3489, 11.7476]	[310, 569]	[14.3792, 20.4354]	[203, 311]
t_{p2}	[3.2519, 7.7395]	[795, 864]	[4.2796, 12.1591]	[522, 681]
t_{p3}	[2.9685, 4.3086]	[871, 1552]	[5.4381, 8.2076]	[536, 774]
Estimators	$\rho = 0.7$		$\rho = 0.6$	
	MSE	PRE	MSE	PRE
$\bar{y}_{[n]N}$	[33.8702, 68.0986]	[100, 100]	[35.8566, 67.8329]	[100, 100]
$\bar{y}_{rN}$	[26.6266, 41.3841]	[127, 165]	[31.8830, 52.6924]	[112, 129]
$\bar{y}_{pN}$	[81.3494, 223.5612]	[30, 42]	[82.8448, 211.6999]	[32, 43]
$\bar{y}_{regN}$	[22.0309, 32.2804]	[154, 211]	[25.1446, 38.9294]	[143, 174]
$\bar{y}_{exprN}$	[25.2190, 38.6479]	[134, 176]	[28.4930, 44.1718]	[126, 154]
$\bar{y}_{exppN}$	[52.5804, 129.7364]	[52, 64]	[53.9739, 123.6756]	[55, 66]
t_{p1}	[21.9881, 32.1936]	[154, 212]	[25.0939, 38.8304]	[143, 175]
t_{p2}	[4.3620, 13.3893]	[509, 776]	[4.7355, 14.1430]	[480, 757]
t_{p3}	[8.2888, 13.2007]	[409, 516]	[9.9709, 16.7169]	[360, 406]

Table 5 MSEs and PREs of the recommended and existing estimator under NMRSS for even case

Estimators	$\rho = 0.9$		$\rho = 0.8$	
	MSE	PRE	MSE	PRE
$\bar{y}_{[n]N}$	[16.6856, 49.0243]	[100, 100]	[19.6011, 46.4910]	[100, 100]
$\bar{y}_{rN}$	[6.8564, 9.9145]	[243, 494]	[12.4269, 19.1244]	[158, 243]
$\bar{y}_{pN}$	[50.6041, 182.1930]	[27, 33]	[52.2026, 171.3741]	[27, 38]
$\bar{y}_{regN}$	[6.2676, 8.8094]	[266, 556]	[10.8760, 15.4188]	[180, 302]
$\bar{y}_{exprN}$	[8.7599, 17.7120]	[190, 277]	[12.8356, 20.6181]	[153, 225]
$\bar{y}_{exppN}$	[30.6337, 103.8513]	[47, 54]	[32.7235, 96.7430]	[48, 60]
t_{p1}	[6.2610, 8.7838]	[267, 558]	[10.8639, 15.3859]	[180, 302]
t_{p2}	[2.1239, 5.8077]	[786, 844]	[2.6848, 9.1361]	[509, 730]
t_{p3}	[2.2015, 3.2341]	[758, 1516]	[3.9952, 6.1923]	[491, 751]
Estimators	$\rho = 0.7$		$\rho = 0.6$	
	MSE	PRE	MSE	PRE
$\bar{y}_{[n]N}$	[23.5002, 49.9260]	[100, 100]	[25.0812, 49.5001]	[100, 100]
$\bar{y}_{rN}$	[18.9892, 30.1761]	[124, 165]	[22.6217, 38.5168]	[111, 129]
$\bar{y}_{pN}$	[52.1644, 163.5474]	[31, 45]	[53.1778, 154.5742]	[32, 47]
$\bar{y}_{regN}$	[16.4401, 24.2585]	[143, 206]	[18.8473, 29.1725]	[133, 170]
$\bar{y}_{exprN}$	[18.2256, 28.3171]	[129, 176]	[20.6468, 32.2471]	[121, 154]
$\bar{y}_{exppN}$	[34.8132, 95.0028]	[53, 68]	[35.9249, 90.2758]	[55, 70]
t_{p1}	[16.4191, 24.2121]	[143, 206]	[18.8220, 29.1197]	[133, 170]
t_{p2}	[2.7098, 10.0417]	[497, 867]	[2.9217, 10.6288]	[466, 858]
t_{p3}	[5.8993, 9.9317]	[398, 503]	[7.0985, 12.5925]	[353, 393]

Table 6 PREs of the NMRSS estimators over estimators under NSRS

$n = 12$ Estimators	$\rho = 0.9$ PRE	$\rho = 0.8$ PRE	$\rho = 0.7$ PRE	$\rho = 0.6$ PRE
$\bar{y}_{[n]N}$	[119, 209]	[122, 187]	[118, 160]	[119, 152]
$\bar{y}_{rN}$	[118, 118]	[122, 130]	[117, 125]	[123, 140]
$\bar{y}_{pN}$	[121, 250]	[120, 233]	[121, 226]	[120, 210]
$\bar{y}_{regN}$	[116, 119]	[125, 126]	[117, 118]	[123, 127]
$\bar{y}_{exprN}$	[117, 151]	[124, 139]	[116, 119]	[120, 126]
$\bar{y}_{exp pN}$	[120, 237]	[121, 218]	[120, 201]	[120, 186]
t_{p1}	[115, 119]	[125, 125]	[117, 118]	[123, 127]
t_{p2}	[124, 203]	[120, 230]	[124, 264]	[125, 256]
t_{p3}	[119, 120]	[124, 132]	[118, 130]	[124, 144]

$n = 16$ Estimators	$\rho = 0.9$ PRE	$\rho = 0.8$ PRE	$\rho = 0.7$ PRE	$\rho = 0.6$ PRE
$\bar{y}_{[n]N}$	[122, 240]	[125, 205]	[121, 171]	[122, 161]
$\bar{y}_{rN}$	[122, 122]	[125, 135]	[121, 132]	[126, 148]
$\bar{y}_{pN}$	[124, 300]	[123, 277]	[124, 260]	[123, 242]
$\bar{y}_{regN}$	[119, 122]	[128, 129]	[120, 121]	[126, 130]
$\bar{y}_{exprN}$	[119, 162]	[128, 143]	[118, 124]	[123, 130]
$\bar{y}_{exp pN}$	[123, 281]	[124, 251]	[123, 224]	[123, 207]
t_{p1}	[119, 122]	[127, 129]	[120, 121]	[126, 130]
t_{p2}	[128, 240]	[123, 283]	[128, 323]	[129, 318]
t_{p3}	[123, 125]	[128, 139]	[121, 141]	[128, 156]

Table 7 PREs of the NMRSS estimators over estimators under NRSS

$n = 12$ Estimators	$\rho = 0.9$ PRE	$\rho = 0.8$ PRE	$\rho = 0.7$ PRE	$\rho = 0.6$ PRE
$\bar{y}_{[n]N}$	[109, 114]	[102, 113]	[107, 110]	[107, 109]
$\bar{y}_{rN}$	[107, 108]	[102, 104]	[108, 110]	[103, 107]
$\bar{y}_{pN}$	[101, 123]	[101, 119]	[101, 120]	[101, 114]
$\bar{y}_{regN}$	[106, 107]	[103, 105]	[106, 108]	[101, 106]
$\bar{y}_{exprN}$	[102, 107]	[104, 106]	[106, 109]	[101, 104]
$\bar{y}_{exp pN}$	[102, 120]	[101, 117]	[109, 115]	[1011, 111]
t_{p1}	[106, 107]	[103, 105]	[106, 108]	[101, 106]
t_{p2}	[102, 121]	[109, 119]	[102, 128]	[103, 122]
t_{p3}	[108, 109]	[102, 106]	[102, 107]	[103, 110]

$n = 16$ Estimators	$\rho = 0.9$ PRE	$\rho = 0.8$ PRE	$\rho = 0.7$ PRE	$\rho = 0.6$ PRE
$\bar{y}_{[n]N}$	[109, 113]	[101, 112]	[101, 105]	[107, 109]
$\bar{y}_{rN}$	[106, 109]	[102, 104]	[101, 108]	[103, 107]
$\bar{y}_{pN}$	[101, 123]	[101, 119]	[101, 119]	[101, 114]
$\bar{y}_{regN}$	[105, 108]	[103, 104]	[101, 108]	[101, 106]
$\bar{y}_{exprN}$	[101, 107]	[103, 106]	[101, 108]	[101, 105]
$\bar{y}_{exp pN}$	[109, 120]	[102, 117]	[101, 113]	[101, 111]
t_{p1}	[105, 108]	[103, 104]	[101, 108]	[101, 106]
t_{p2}	[102, 122]	[110, 119]	[102, 129]	[102, 122]
t_{p3}	[109, 111]	[102, 105]	[102, 107]	[103, 110]

The comparison between classical MRSS and NMRSS using MSEs and PREs is provided in Tables 8 and 9. Tables 8 and 9 demonstrate that the MSEs of the suggested estimators obtained through classical MRSS are between the lower and higher value of MSE obtained using NMRSS, and the same goes for the PREs, indicating that the latter method is more effective than the former.

The study highlights that classical ranked set sampling is ill-suited for dealing with vague or indeterminate data. NMRSS proves superior for estimating uncertain or interval data. The tables present dependable results for neutrosophic data compared to classical results.

9 Conclusion

This study aims to tackle the challenges posed by ambiguous or indeterminate data within the realm of classical statistics. Through the utilization of MRSS, an enhancement over SRS and RSS, the study addresses the complexities associated with expensive and asymmetric data. Given the absence of prior research addressing this specific issue, our work represents a novel and significant advancement in survey sampling methodologies, particularly in dealing with imprecise data featuring asymmetric distributions.

In this research paper, we have proposed a new modification of RSS: the very first neutrosophic median ranked set method in sampling theory. We've put forth some enhanced NMRSS estimators designed for estimating population means while making use of auxiliary information. To assess their accuracy, we calculated both bias and MSE for these suggested estimators, focusing on first-order approximations. We compared our recommended estimators against existing ones using a natural population data on natural growth rates and a simulated population.

Through a combination of numerical illustrations and simulation studies, we've found compelling evidence that our proposed estimators outperform existing ones within the framework of NMRSS. Among these estimators, t_{p2} emerged as the top performer. It's important to note that the sensitivity analysis of our recommended estimators under NMRSS mirrors that of classical MRSS. The MSE and PRE of the recommended estimator decrease and increase, respectively, with the increase in values of sample sizes and correlation coefficients.

Moreover, a comparison between the recommended estimators under NMRSS and the estimators under NSRS and NRSS revealed that NMRSS is a more effective alternative to NSRS and NRSS, much like classical MRSS to classical SRS and classical RSS. The MSEs of the proposed estimators derived from classical MRSS fall within the range between the lower and higher MSE values obtained through NMRSS. Similarly, the PREs exhibit a similar pattern. These findings

Table 8 MSEs and PREs of the estimators (neutrosophic vs classical) for odd case

$\rho = 0.9$				
Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[25.8558, 66.8489]	30.3481	[100, 100]	100
$\bar{y}_{rN}$	[9.4335, 13.6782]	10.6122	[274, 489]	286
$\bar{y}_{pN}$	[82.3835, 248.6207]	97.9875	[27, 31]	31
$\bar{y}_{regN}$	[8.3641, 11.7963]	9.2976	[309, 567]	326
$\bar{y}_{exprN}$	[12.6315, 24.1884]	14.4922	[205, 276]	209
$\bar{y}_{exppN}$	[49.1065, 141.6597]	58.1799	[47, 53]	52
t_{p1}	[8.3489, 11.7476]	9.2816	[310, 569]	327
t_{p2}	[3.2519, 7.7395]	3.8547	[795, 864]	787
t_{p3}	[2.9685, 4.3086]	3.3317	[871, 1552]	911
$\rho = 0.8$				
Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[29.1643, 63.5223]	35.6477	[100, 100]	100
$\bar{y}_{rN}$	[17.2001, 26.2012]	20.5729	[170, 242]	173
$\bar{y}_{pN}$	[83.7130, 234.4552]	98.9972	[27, 35]	36
$\bar{y}_{regN}$	[14.4049, 20.4974]	17.6304	[202, 310]	202
$\bar{y}_{exprN}$	[17.8592, 28.1602]	22.0759	[163, 226]	161
$\bar{y}_{exppN}$	[51.1156, 132.2873]	61.2881	[48, 57]	58
t_{p1}	[14.3792, 20.4354]	17.6007	[203, 311]	203
t_{p2}	[4.2796, 12.1591]	4.8312	[522, 681]	738
t_{p3}	[5.4381, 8.2076]	6.4180	[536, 774]	555
$\rho = 0.7$				
Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[33.8702, 68.0986]	40.3408	[100, 100]	100
$\bar{y}_{rN}$	[26.6266, 41.3841]	29.8109	[127, 165]	135
$\bar{y}_{pN}$	[81.3494, 223.5612]	99.3064	[30, 42]	41
$\bar{y}_{regN}$	[22.0309, 32.2804]	24.9118	[154, 211]	162
$\bar{y}_{exprN}$	[25.2190, 38.6479]	29.0213	[134, 176]	139
$\bar{y}_{exppN}$	[52.5804, 129.7364]	63.7691	[52, 64]	63
t_{p1}	[21.9881, 32.1936]	24.8667	[154, 212]	162
t_{p2}	[4.3620, 13.3893]	5.2121	[509, 776]	774
t_{p3}	[8.2888, 13.2007]	9.2138	[409, 516]	438
$\rho = 0.6$				
Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[35.8566, 67.8329]	44.1928	[100, 100]	100
$\bar{y}_{rN}$	[31.8830, 52.6924]	38.5595	[112, 129]	115
$\bar{y}_{pN}$	[82.8448, 211.6999]	98.3161	[32, 43]	45
$\bar{y}_{regN}$	[25.1446, 38.9294]	31.2729	[143, 174]	141
$\bar{y}_{exprN}$	[28.4930, 44.1718]	35.3149	[126, 154]	125
$\bar{y}_{exppN}$	[53.9739, 123.6756]	65.1932	[55, 66]	68
t_{p1}	[25.0939, 38.8304]	31.2119	[143, 175]	142
t_{p2}	[4.7355, 14.1430]	5.3729	[480, 757]	823
t_{p3}	[9.9709, 16.7169]	11.8420	[360, 406]	373

Table 9 MSEs and PREs of the estimators (neutrosophic vs classical) for even case

$\rho = 0.9$ Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[16.68563, 49.02427]	19.51716	[100, 100]	100
$\bar{y}_{rN}$	[6.85639, 9.91454]	7.66664	[243, 494]	255
$\bar{y}_{pN}$	[50.60409, 182.193]	59.9208	[27, 33]	33
$\bar{y}_{regN}$	[6.26758, 8.80943]	6.96706	[266, 556]	280
$\bar{y}_{exprN}$	[8.75986, 17.71203]	10.02276	[190, 277]	195
$\bar{y}_{expPN}$	[30.63371, 103.8513]	36.14984	[47, 54]	54
t_{p1}	[6.26095, 8.78384]	6.96016	[267, 558]	280
t_{p2}	[2.12389, 5.80765]	2.48653	[786, 844]	785
t_{p3}	[2.20148, 3.23414]	2.45599	[758, 1516]	795
$\rho = 0.8$ Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[19.60108, 46.49099]	23.88907	[100, 100]	100
$\bar{y}_{rN}$	[12.42685, 19.12443]	14.83691	[158, 243]	161
$\bar{y}_{pN}$	[52.20264, 171.3741]	61.72381	[27, 38]	39
$\bar{y}_{regN}$	[10.87602, 15.41875]	13.21414	[180, 302]	181
$\bar{y}_{exprN}$	[12.83555, 20.61814]	15.76516	[153, 225]	152
t_{p1}	[10.86386, 15.38587]	13.20018	[180, 302]	181
t_{p2}	[2.68475, 9.13607]	3.02011	[509, 730]	791
t_{p3}	[3.99515, 6.19232]	4.65817	[491, 751]	513
$\rho = 0.7$ Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[23.50022, 49.92603]	27.59529	[100, 100]	100
$\bar{y}_{rN}$	[18.98922, 30.17608]	21.4977	[124, 165]	128
$\bar{y}_{pN}$	[52.16444, 163.5474]	62.57538	[31, 45]	44
$\bar{y}_{regN}$	[16.4401, 24.25845]	18.72936	[143, 206]	147
$\bar{y}_{exprN}$	[18.22557, 28.31713]	20.93618	[129, 176]	132
$\bar{y}_{expPN}$	[34.81318, 95.00277]	41.47502	[53, 68]	67
t_{p1}	[16.41913, 24.21205]	18.70726	[143, 206]	148
t_{p2}	[2.70979, 10.04169]	3.23187	[497, 867]	854
t_{p3}	[5.89925, 9.93174]	6.65863	[398, 503]	414
$\rho = 0.6$ Estimators	MSE		PRE	
	Neutrosophic	Classical	Neutrosophic	Classical
$\bar{y}_{[n]N}$	[25.08118, 49.50014]	30.90622	[100, 100]	100
$\bar{y}_{rN}$	[22.62167, 38.51675]	27.72227	[111, 129]	111
$\bar{y}_{pN}$	[53.17782, 154.5742]	63.02822	[32, 47]	49
$\bar{y}_{regN}$	[18.8473, 29.17245]	23.55219	[133, 170]	131
$\bar{y}_{exprN}$	[20.64679, 32.24711]	25.69699	[121, 154]	120
$\bar{y}_{expPN}$	[35.92486, 90.27583]	43.34997	[55, 70]	71
t_{p1}	[18.82196, 29.1197]	23.52152	[133, 170]	131
t_{p2}	[2.9217, 10.62878]	3.31264	[466, 858]	933
t_{p3}	[7.09854, 12.59245]	8.48045	[353, 393]	364

suggest that the suggested method surpasses the effectiveness of classical approaches. Our study underscores the efficiency and reliability of NMRSS for handling neutrosophic data, with our proposed NMRSS delivering superior mean estimations compared to existing methods.

Based on the numerical illustrations and simulation studies we've conducted, it's reasonable to recommend the use of our proposed estimators over the alternatives presented in this paper in various real-world scenarios, spanning fields like agriculture, mathematics, biology, poultry farming, economics, commerce, and the social sciences.

Furthermore, given the limited availability of neutrosophic MRSS estimators, there's ample room for further exploration. Building upon this study, we can consider defining variations of NRSS, such as unbalanced NRSS, quantile NRSS, extreme NRSS, double NRSS, and percentile NRSS, akin to what exists in classical RSS. Additionally, we can explore the replacement of our proposed estimators with alternative methods or estimators. Expanding beyond sampling theory, further research avenues in statistics, encompassing fields like control charts, inference, reliability analysis, non-parametric estimation, hypothesis testing, and some other fields of science, present promising opportunities for exploration.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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